The performance curves in Fig. 2 show that the combination of an 11.3 m^3 sidewall muffler and an active attenuator operating in the 25-125 Hz band will provide the required attenuation. Detailed calculations show that about 20% of the acoustic energy to be attenuated lies in the 25-125 Hz band; and the round-trip passage through the sidewall muffler attenuates that band by a factor of 0.007 to 0.022 W.

Thus, an active attenuator emitting only a few hundredths of a watt would enable a reduction in the passive muffler volume of about 35%. Since even a simple piston speaker can achieve 4% efficiency for narrow frequency ranges, only about 1 W of power would be required to drive the four speakers. If the laser were operated at a few atmospheres pressure, the characteristic acoustic impedance of the laser gas would increase, with a corresponding improvement in speaker efficiency. Another possibility for improving speaker efficiency would be to use an acoustic horn, providing its presence did not cause an unacceptable perturbation in the gas flow.

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Correction of Stiffness Matrix Using Vibration Tests

Menahem Baruch*
Technion—Israel Institute of Technology, Haifa, Israel

IN a recent Note[1] Wei reobtains the corrected stiffness matrix given in Refs. 2-5. It is interesting that, by algebraic manipulations, Wei succeeds in eliminating the Lagrange multiplier λ_y from the derivation. In this way the need to calculate λ_y is avoided. Unger and Zalmanovich[6] proposed different constraints from those used in Ref. 3. These constraints constitute of the requirement that the corrected measured modes must be orthogonal between themselves and in addition must also be orthogonal to the unmeasured modes. In this way, Zalmanovich[7] succeeds in obtaining the necessary Lagrange multipliers and the corrected stiffness matrix without any additional assumptions.

In Ref. 3 the Lagrange multiplier λ_y was obtained by making the assumption that the matrix $\lambda_y^t MX$ is symmetric. In his Note,[1] Wei makes the remark that this assumption "is not always true in general and is hard to understand from a physical point of view." In what follows, the meaning of the assumption is given and it is shown that it is a permissible assumption and therefore true.

The corrected stiffness matrix Y was obtained in Refs. 2-5 by minimization of the weighted Euclidean norm

$$g = \tfrac{1}{2} \| N^{-1} (Y-K) N^{-1} \| \tag{1}$$

where

$$N = M^{\frac{1}{2}} \tag{2}$$

M is the mass matrix, Y $(n \times n)$ the corrected stiffness matrix, and K $(n \times n)$ a given stiffness matrix.

The corrected stiffness matrix Y must satisfy the following constraints[2-5]

$$YX = MX\Omega^2 \tag{3}$$

and

$$Y = Y^t \tag{4}$$

where Ω^2 $(m \times m)$ represents the measured frequencies and X $(n \times m)$ the already orthogonalized incomplete set of measured modes[2-5]

$$X^t MX = I \tag{5}$$

The constraints of Eqs. (3) and (4) were incorporated in Eq. (1) by using Lagrange multipliers. Minimization of the cost function g, with respect to y_{ij} and elimination of the Lagrange multiplier connected with Eq. (4) gave[2-5]

$$Y = K - M\lambda_y X^t M - MX\lambda_y^t M \tag{6}$$

where λ_y $(n \times m)$ is the Lagrange multiplier connected with the constraint of Eq. (3). Using this last constraint the following equation was obtained

$$MX\Omega^2 = KX - M\lambda_y - MX\lambda_y^t MX \tag{7}$$

Here the following crucial assumption was made

$$\lambda_y^t MX = X^t M\lambda_y \tag{8}$$

The assumption of Eq. (8) was used in Ref. 3 to obtain the corrected stiffness matrix [Ref. 3, Eq. (23)]

$$Y = K - KXX^t M - MXX^t K + MXX^t KXX^t M + MX\Omega^2 X^t M \tag{9}$$

In Ref. 3 it was shown that although λ_y which satisfies Eq. (7) is not unique, the corrected stiffness matrix obtained from any such λ_y is the same and is given in Eq. (9). In other words, any assumption for λ_y that does not violate Eqs. (3), (7) and itself is true and can be used to obtain Y. More than that, Eq. (8) has also a clear physical and mathematical meaning.

In the minimization process in Ref. 3 the constraint of Eq. (3) was taken formally to represent mn independent constraints. However, due to the fact that the mode shapes X are orthogonal, one obtains

$$X_i^t YX_j = 0 \qquad i \neq j \tag{10}$$

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*Professor, Department of Aeronautical Engineering. Member AIAA.

Taking into account Eq. (4), Eq. (10) represents $m(m-1)/2$ linear connections between the constraints given in Eq. (3). In other words, Eq. (3) represents only $nm-m(m-1)/2$ independent equations and therefore $\lambda_y(m\times n)$ must have exactly the same number of independent coefficients. It is interesting to note that when X contains the full set of mode shapes $(m=n)$, the number of independent equations in Eq. (3) becomes equal to the number of independent variables, $n(n+1)/2$, which is in full agreement with the physical situation. In this case one obtains from Eq. (9)[2,3] or directly from Eq. (3)

$$Y_{\text{full}}=MX_{\text{full}}\Omega^2_{\text{full}}X^t_{\text{full}}M \tag{11}$$

Now, any set of connections between the coefficients of λ_y which reduces the number of independent coefficients to the number of independent equations in Eq. (3) and does not violate Eq. (10) is permissible. Equation (8) represents such a permissible set. The mathematical and physical meaning of Eq. (8) is now clear: it represents $m(m-1)/2$ linear connections between the coefficients of λ_y which take into account the orthogonality conditions between the mode shapes. Equation (8) makes the problem well defined.

Conclusion

It was shown that the assumption $\lambda_y^t MX = X^t M\lambda_y$ used in Ref. 3 to obtain a corrected stiffness matrix has clear mathematical and physical meaning. It reduces the number of coefficients in the Lagrange multiplier λ_y to the number of independent equations in full consideration with the orthogonality conditions.

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Generalized Substructure Coupling Procedure for Damped Systems

Roy R. Craig Jr.* and Yung-Tseng Chung†
The University of Texas, Austin, Tex.

Introduction

MANY previous papers have discussed the use of substructure coupling, in particular the version of substructure coupling referred to as component mode synthesis, for solving large structural dynamics problems. Only a very limited number of these works have treated damped structures, and the usual approach has been to assume "proportional damping" and to assume the structures to be lightly damped. However, there are important instances in which these damping assumptions are invalid. For example, structures with active control systems, with concentrated dampers, or with rotating parts fall in this category. Also, it is quite common for structural modes that are measured experimentally to be complex modes, i.e., modes which do not satisfy the proportional damping assumption. In Ref. 1 Hasselman and Kaplan extended the Craig-Bampton form of the classical Hurty method of substructure coupling to systems with nonproportional damping by using fixed-interface complex substructure modes. A generalized substructure coupling procedure for damped structures was presented in Ref. 2. The present Note summarizes the findings of that work by presenting a generalized substructure coupling procedure for damped systems. The essential ingredients of the method are 1) a Hamiltonian first-order differential equation formulation is used, and 2) a generalized coupling procedure is employed, permitting all significant types of constraints and coordinate transformations to be invoked. An alternative state vector formulation is presently under study.

Theoretical Formulation

The name "component mode synthesis" is applied to substructure coupling procedures in which the motion of each substructure is represented by a selected set of component modes. Equations of interface compatibility are employed to obtain an independent set of system equations of motion. In the present Note, the substructure equations are written in the first order form

$$a\dot{y}+by=f \tag{1}$$

where

$$a=\begin{bmatrix}0 & I\\ I & c\end{bmatrix},\quad b=\begin{bmatrix}-m^{-1} & 0\\ 0 & k\end{bmatrix},\quad y=\begin{Bmatrix}p\\ x\end{Bmatrix},\quad f=\begin{Bmatrix}0\\ f_x\end{Bmatrix} \tag{2}$$

where x is the substructure physical coordinate vector, $p=m\dot{x}$ the substructure momentum vector, m the substructure mass matrix, c the substructure damping matrix, and k the substructure stiffness matrix. [The form of a and b in Eq. (2) is selected so that the matrices will be symmetric when c is symmetric.] Substructure modes, which may be complex, are obtained by solving the homogeneous form of Eq. (1) by assuming a solution of the form

$$y=\psi e^{\lambda t} \tag{3}$$

For underdamped systems the eigenvalues λ_r and eigenvectors ψ_r occur in complex conjugate pairs. Let Ψ be the collection of substructure modal vectors. Then the modal synthesis coordinate transformation is given by

$$y=\Psi z \tag{4}$$

where z is the vector of substructure modal coordinates.

When a structure consists of several substructures, the coordinates x will be subject to interface compatibility conditions (constraints). Let X, P, and Y be the collection of all substructure x, p, and y coordinates, respectively. Thus,

$$P=M\dot{X} \tag{5}$$

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*Professor, Aerospace Engineering and Engineering Mechanics. Member AIAA.

†Graduate Student, Engineering Mechanics.